Extended
Mathematics
for Cambridge **IGCSE** THIRD EDITION

OXFORD

Model answers

Doing questions from past papers is a great way to revise. It gives you practice in applying what you have learned, in different contexts.

Here are some model answers to the kinds of questions that come up quite often in exams, and some comments on how the answer was found.

Question 1

Cambridge IGCSE Mathematics 0580 Paper 21 Q4 November 2009

Simplify $\frac{5}{8}x^{\frac{3}{2}} \div \frac{1}{2}x^{-\frac{5}{2}}$. *[2]*

Model Answer

$1\frac{1}{4}x^4$

Comment

For any question requiring simplification you must deal with each part in turn.

Numbers:

$\frac{5}{8} \div \frac{1}{2} = \frac{5}{8} \times \frac{2}{1} = \frac{10}{8} = \frac{5}{4}$

Remember

Dividing fractions means multiplying by the reciprocal

Letters:

$x^{\frac{3}{2}} \div x^{-\frac{5}{2}} = x^{\frac{3}{2} - \frac{-5}{2}} = x^{\frac{3}{2}+\frac{5}{2}} = x^{\frac{8}{2}} = x^4$

Remember

Division means subtract the indicies

So the answer is

$1\frac{1}{4}x^4$

Question 2

Cambridge IGCSE Mathematics 0580 Paper 21 Q6 November 2009

$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -8 & -4 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 7 & 1 \\ 0 & -5 \end{pmatrix}$

Calculate the value of $5|\mathbf{A}| + |\mathbf{B}|$, *where* $|\mathbf{A}|$ *and* $|\mathbf{B}|$ *are the determinants of* $\mathbf{A}$ *and* $\mathbf{B}$. *[2]*

Model Answer

$|\boldsymbol{A}| = (0 \times -4) - (-8 \times 1)$
$= 0 + 8$
$= 8$

$|\boldsymbol{B}| = (7 \times -5) - (1 \times 0)$
$= -35 - 0$
$= -35$

So, $5|\boldsymbol{A}| + |\boldsymbol{B}| = 5 \times 8 - 35$
$= 40 - 35$
$= 5$

Comment

Matrix questions can appear much more complicated than they actually are. This question should be very straightforward providing you know the method for working out the determinant.

There are two main points to watch:

1. The negative signs can lead to easy errors.
   Remember that when multiplying two negative numbers, the answer is positive.
2. Multiplying by zero always results in a zero.
   This is easy to forget in an exam situation.

Question 3

Cambridge IGCSE Mathematics 0580 Paper 21 Q10 November 2009

The braking distance, d, of a car is directly proportional to the square of its speed, v.

When d = 5, v = 10.
Find d when v = 70. *[3]*

Model Answer

$d \propto v^2$

This means that:

$d = kv^2$

Substituting:

$k = \frac{d}{v^2} = \frac{5}{100} = \frac{1}{20}$

So:

$d = \frac{1}{20}v^2$

If $v = 70$,

$d = \frac{1}{20} \times 4900$

$d = 245$

Comment

Proportion questions are often poorly done.
As soon as you see the phrase 'is proportional to …' you need to replace the proportion part by '= k multiplied by …'
The 'square of the speed, v' simply means v^2.
So, 'd is directly proprtional to the square of the speed, v' means:

$$d = kv^2$$

From here, you just need to do some simple algebra so the first step is key!
Inverse questions are not much more difficult really. Just replace the inverse part by '= k multiplied by 1/…'

For example, y is inversely proportional to x means:

$$y = \frac{k}{x}$$

Question 4

Cambridge IGCSE Mathematics 0580 Paper 21 Q14 November 2009

Zainab borrows \$198 from a bank to pay for a new bed.
The bank charges compound interest at 1.9% per month.
Calculate how much interest she owes at the end of 3 months.
Give your answer correct to 2 decimal places. *[3]*

Model Answer

1.9% = 0.019
So compound interest = 1.019

$198 \times 1.019^3 = 209.50$

The interest paid is therefore:
209.50 – 198 = \$11.50

Comment

There is a simple technique to compound interest questions.
Work out the interest rate as a decimal and then add this to 1 to give the multiplying factor.
This question asked for interest for 3 months so simply use the multiplying factor three times. This is the same as raising the multiplying factor to the power of 3.
In this case we subtract the amount borrowed to find the interest paid.

Question 5

Cambridge IGCSE Mathematics 0580 Paper 22 Q6 June 2009

In 2005 there were 9 million bicycles in Beijing, correct to the nearest million.
The average distance travelled by each bicycle in one day was 6.5 km correct to one decimal place.
Work out the upper bound for the total distance travelled by all the bicycles in one day. *[2]*

Model Answer

9 million correct to nearest million gives an upper bound of 9 500 000.
6.5 km correct to 1 d.p. gives an upper bound of 6.55 km.
$9\,500\,000 \times 6.55 = 62\,225\,000$ km

Comment

Questions on upper and lower bounds are almost guaranteed to come up and they should present no problems if you have revised the topic properly.

Take each part of the information in turn:

- 'nearest million' means half the million to give 500 000. The upper bound (UB) is found by adding this to the original amount.
- 'correct to 1 d.p.' means we need a number with 2 d.p. that would change to 6.5 if rounded to 1d.p.
  This would give an UB of 6.549 999… which we round to give 6.55

The final answer is found by multiplying these answers together. Although you may think about *recurring* decimals to help you work out the answer, ALWAYS use the rounded form otherwise things will get a little too complicated!

Question 6

Cambridge IGCSE Mathematics 0580 Paper 22 Q18 June 2009

Two similar vases have heights which are in the ratio 3:2.

a) The volume of the larger vase is 1080 cm^3.
Calculate the volume of the smaller vase. *[2]*

b) The surface area of the smaller vase is 252 cm^2.
Calculate the surface area of the larger vase. *[2]*

Model Answer

The ratio of the heights is 3:2
So the larger vase is 1.5 times bigger than the small vase.
The small vase is $\frac{2}{3}$ of the size of the larger vase.
This means that the volume of the smaller vase will be $\left(\frac{2}{3}\right)^3 \times 1080 = 320\,\text{cm}^2$
The area of the larger vase will be $(1.5)^2 \times 252 = 567\,\text{cm}^3$

Comment

Questions on similar shapes are often misunderstood.
As soon as you see the phrase 'similar shape' this should signal 'think powers'!
Similar *areas* mean *square* the factor.
Similar *volumes* mean *cube* the factor.

If you think of the units:

- area is in cm^2 or 'cm squared'
- volume is in cm^3 or 'cm cubed'

So it should be easy to remember!

Question 7

Cambridge IGCSE Mathematics 0580 Paper 22 Q8 June 2008

Simplify $(16x^4)^{\frac{3}{4}}$. *[2]*

Model Answer

Take the power into both parts of the bracket:

$$16^{\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^3 = 2^3 = 8$$

$$(x^4)^{\frac{3}{4}} = x^{\frac{(4\times3)}{4}} = x^3$$

So, the answer is $8x^3$

Comment

The main point to remember is to raise *both* parts of the bracket to the stated power.

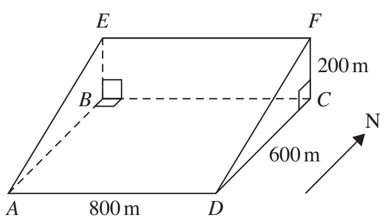
This means work out the numbers first and then deal with the algebra.

If you can't deal with $16^{\frac{3}{4}}$ on your calculator split it up into $16^{\left(\frac{1}{4}\right)\times 3}$ and so it becomes $\left(16^{\frac{1}{4}}\right)^3$. This makes it easier to work out.

For the algebra, remember the rule: $(x^a)^b = x^{ab}$

Question 8

Cambridge IGCSE Mathematics 0580 Paper 22 Q21 June 2008



NOT TO SCALE

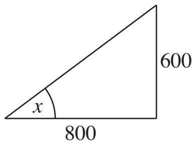
ABCD, BEFC and AEFD are all rectangles.
ABCD is horizontal, BEFC is vertical and AEFD represents a hillside.
AF is a path on the hillside.
AD = 800 m, DC = 600 m and CF = 200 m.

a) *Calculate the angle that the path AF makes with ABCD.* *[5]*

b) *In the diagram D is due south of C.*
Jasmine walks down the path from F to A in bad weather.
She cannot see the path ahead. The compass bearing she must use is the bearing of A from C.
Calculate this bearing. *[3]*

Model Answer

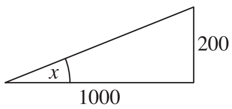
a)



Pythagoras' theorem gives the longest side as:

$\sqrt{600^2 + 800^2} = 1000$

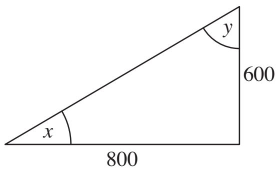
Trigonometry gives the angle as:



$\tan x = \frac{200}{1000}$

$x = \tan^{-1}(0.2) = 11.3°$

b)



$\tan x = \frac{600}{800}$

$x = \tan^{-1}\left(\frac{3}{4}\right) = 37°$

$y = 180 - 90 - 37 = 53°$

Bearing of A from C is 180 + 53 = 233°

Comment

a) Questions on planes and angles can be confusing. The easiest way is to draw a diagram and make sure you know exactly which angle you're meant to be finding.

This question required the use of both Pythagoras' theorem and trigonometry – a common theme in the extended paper.
Pythagoras should be fine – just remember to square root at the end!
In trigonometry questions, be sure to learn the three formulae:

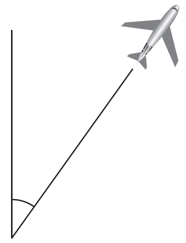
$\sin x = \frac{\text{opp}}{\text{hyp}}$ A good idea is to write these three formulae down as soon as you start the exam

$\cos x = \frac{\text{adj}}{\text{hyp}}$

$\tan x = \frac{\text{opp}}{\text{adj}}$

In this case, we used the tan formula because we were given the opposite and adjacent sides.

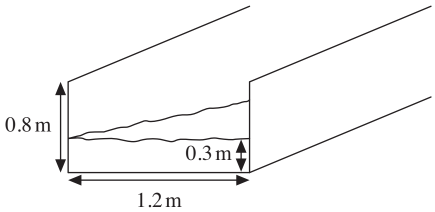
b) Again, a good diagram helps work out which angle is needed. The best way to do this was to work out x, then use the angles of a triangle to give y. Then simply give this angle as a bearing.



Remember
Bearings are always measured clockwise from the North line.

Question 9

Cambridge IGCSE Mathematics 0580 Paper 4 Q7 June 2007



NOT TO SCALE

The diagram shows water in a channel.
This channel has a rectangular cross-section, 1.2 metres by 0.8 metres.

a) *When the depth of water is 0.3 metres, the water flows along the channel at 3 metres/**minute**.*
Calculate the number of cubic metres which flow along the channel in one hour. *[3]*

b) *When the depth of water in the channel increases to 0.8 metres, the water flows at 15 metres/minute.*
Calculate the percentage increase in the number of cubic metres which flow along the channel in one hour. *[4]*

c) *The water comes from a cylindrical tank.*
*When 2 cubic metres of water leave the tank, the level of water in the tank goes down by 1.3 **millimetres**.*
*Calculate the radius of the tank, in **metres**, correct to one decimal place.* *[4]*

d) *When the channel is empty, its **interior** surface is repaired.*
This costs \$0.12 per square metre. The total cost is \$50.40.
Calculate the length, in metres, of the channel. *[4]*

Model Answer

a) One hour is 60 mins.
Cross-section = $1.2 \times 0.3 = 0.36$ m^2
The flow rate is 3 m/min so in one hour:
$0.36 \times 3 \times 60 = 64.8$ m^3/hr

b) New cross-section: $1.2 \times 0.8 = 0.96$ m^2
15m/min so in one hour: $0.96 \times 15 \times 60 = 864$ m^3/hr

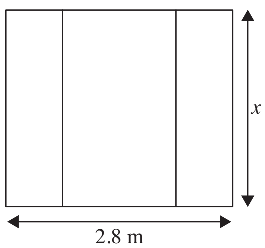
$$\% \text{ increase} = \frac{\text{absolute increase}}{\text{original}} \times 100$$
$$= \frac{(864 - 64.8)}{64.8} \times 100$$
$$= 1233.33\%$$
$$= 1230\% \text{ (3 s.f.)}$$

c) 1.3 mm = 0.0013 metres
We need to solve:
$\pi r^2 h = volume$

So:

$\pi r^2 \times 0.0013 = 2$

$$r^2 = \frac{2}{\pi \times 0.0013}$$
$$r = \sqrt{489.64402879}$$
$r = 22.1$ (3 s.f.)

d) 

The interior surface of the channel is simply a rectangle with the dimensions given above.
If the cost is \$0.12 per m^2 and the total cost is \$50.40 then the area must be:

$$A = \frac{50.4}{0.12}$$
$A = 420$ m^2

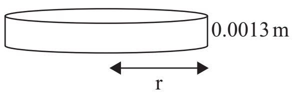
So we have an equation:
$2.8 \times x = 420$
$x = 150$ m

Comment

a) Make sure you are comfortable dealing with units of speed. In this case we want to know the flow in one hour. This is found by multiplying the flow rate in one minute by 60.
Remember there are 60 minutes in one hour, not 100!
The volume is found by simply multiplying the area of the cross-section by the flow rate.

b) Make sure you are happy with working out the percentage increase. It is the absolute increase divided by the original value, and then multiplied by 100.
Note this is different to working out the new flow as a percentage of the old flow. This is a good example of where reading the question carefully is crucial in figuring out what the question is really asking.

c) Drawing a quick picture here makes this question a lot less intimidating.



We are given the height and the volume and so it is just a case of working backwards to get the cross sectional area, and hence the radius. Note that we are given the height in mm. Units must be consistent when substituting into a formula so make sure you know how to convert common units.

$$\text{mm} \xrightarrow{\div 10} \text{cm}$$
$$\text{cm} \xrightarrow[\div 100]{} \text{m}$$

d) Again, a diagram is helpful.
The question here relates to the *channel* so don't get confused with the cylinder as in the previous question.
The first step is working out the *area* of the interior. This is found by a simple division of the cost and cost per metre^2.
The next step is working out the *perimeter* of the channel. Again this is a simple addition.
Finally, we use the simple formula *area* = *width* × *height to* give the required answer.